MATH 1920 Formula Packet

Length of Curves

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx \qquad \text{or} \qquad L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Center of Mass

$$\bar{x} = \frac{\int_a^b x[f(x) - g(x)]\, dx}{\int_a^b [f(x) - g(x)]\, dx}, \qquad \bar{y} = \frac{\int_a^b \frac{1}{2}\left[\left(f(x)\right)^2 - \left(g(x)\right)^2\right] dx}{\int_a^b [f(x) - g(x)]\, dx}$$

Trapezoid Rule and Simpson's Rule

Rule	Error
$T = \frac{\Delta x}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$	$\lvert E_T \rvert \le \frac{M(b-a)^3}{12n^2}$, M is an upperbound of $\lvert f''(x) \rvert$ on [a,b]
$S = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$	$\lvert E_S \rvert \le \frac{M(b-a)^5}{180n^4}$, M is an upperbound of $\lvert f^{(4)}(x) \rvert$ on [a,b]

Theorem 5 The following six sequences converge to the limits listed below:

1. $\lim_{n\to\infty} \frac{\ln n}{n} = 0$
2. $\lim_{n\to\infty} \sqrt[n]{n} = 1$
3. $\lim_{n\to\infty} x^{1/n} = 1 \quad (x > 0)$
4. $\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$
5. $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$ (any x)
6. $\lim_{n\to\infty} \frac{x^n}{n!} = 0$ (any x)

In Formulas (3) through (6), x remains fixed as $n \to \infty$

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Tests for Convergence and Divergence

Test	Series	Converges if	Diverges if	Comments
Geometric	$\sum_{n=1}^{\infty} ar^{n-1}$	$\lvert r \rvert < 1$	$\lvert r \rvert \geq 1$	Sum: $S = \frac{a}{1-r}$
n^{th} – Term	$\sum_{n=1}^{\infty} a_n$	----------	$\lim_{n\to\infty} a_n \neq 0$	Cannot be used to show convergence.
Integral	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) > 0$	$\int_1^{\infty} f(x)dx$ converges	$\int_1^{\infty} f(x)dx$ diverges	f is continuous, positive and decreasing
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Direct Comparison	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ $\sum_{n=1}^{\infty} b_n$ diverges	$a_n, b_n > 0$
Ratio Test	$\sum_{n=1}^{\infty} a_n$ positive terms	$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$	$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$	Inconclusive if $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$
Root Test	$\sum_{n=1}^{\infty} a_n$ where $a_n \geq 0$ for $n \geq N$	$\lim_{n\to\infty} \sqrt[n]{a_n} < 1$	$\lim_{n\to\infty} \sqrt[n]{a_n} > 1$	Inconclusive if $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$
Alternating Series	$\sum_{n=1}^{\infty} (-1)^n a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_n \rvert \leq a_{n+1}$

Taylor Series for f, expanded about $x = a$:

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots\cdots + \frac{f^{(k)}(a)}{k!}(x-a)^k + \cdots\cdots$$

Maclaurin Series for f:

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots\cdots + \frac{f^{(k)}(0)}{k!}x^k + \cdots\cdots$$

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Formulas from MATH 1910:

Select Derivatives

1. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$
2. $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$
3. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$
4. $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$
5. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$
7. $\frac{d}{dx}(a^x) = (\ln a) \cdot a^x$

Special Factoring

1. Sum of Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
2. Difference of Cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Select Integrals

1. $\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
2. $\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$
3. $\int \tan(ax)\, dx = \frac{1}{a}\ln|\sec(ax)| + C$
4. $\int \ln(ax)\, dx = x\ln(ax) - x + C$

Select Geometry Formulas

1. Volume of a Sphere: $V = \frac{4}{3}\pi r^3$
2. Surface Area of Sphere: $S = 2\pi r^2 + 2\pi rh$
3. Volume of a Right Circular Cone: $V = \frac{1}{3}\pi r^2 h$
4. Area of a Trapezoid: $A = \frac{(b_1+b_2)\cdot h}{2}$

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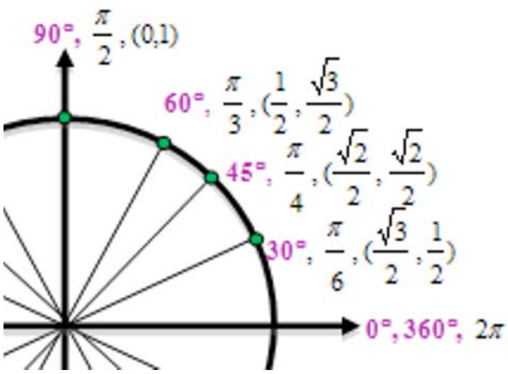
Formulas from MATH 1720:



Area of a Sector, Arc Length, Angular Speed, Linear Speed

$$A = \frac{1}{2}\theta r^2 \qquad s = r\theta \qquad \omega = \frac{\theta}{t} \qquad V = r\omega$$

Sum and Difference Identities

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Double-Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Power-Reducing Identities

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} \qquad \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Product-to-Sum Identities

$$\cos \boldsymbol{a} \cos \boldsymbol{b} = \tfrac{1}{2}\left(\cos(\boldsymbol{a}+\boldsymbol{b}) + \cos(\boldsymbol{a}-\boldsymbol{b})\right)$$

$$\sin \boldsymbol{a} \sin \boldsymbol{b} = \tfrac{1}{2}\left(\cos(\boldsymbol{a}-\boldsymbol{b}) - \cos(\boldsymbol{a}+\boldsymbol{b})\right)$$

$$\sin \boldsymbol{a} \cos \boldsymbol{b} = \tfrac{1}{2}\left(\sin(\boldsymbol{a}+\boldsymbol{b}) + \sin(\boldsymbol{a}-\boldsymbol{b})\right)$$

$$\cos \boldsymbol{a} \sin \boldsymbol{b} = \tfrac{1}{2}\left(\sin(\boldsymbol{a}+\boldsymbol{b}) - \sin(\boldsymbol{a}-\boldsymbol{b})\right)$$

Sum-to-Product Identities

$$\sin\boldsymbol{a} + \sin\boldsymbol{b} = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin\boldsymbol{a} - \sin\boldsymbol{b} = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos\boldsymbol{a} + \cos\boldsymbol{b} = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos\boldsymbol{a} - \cos\boldsymbol{b} = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

Area of a Triangle

$$K = \frac{1}{2}bc\cdot\sin A = \frac{1}{2}ab\cdot\sin C = \frac{1}{2}ac\cdot\sin B$$

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \tfrac{1}{2}(a+b+c)$$

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Formulas from MATH 1720:

Angle Between Two Vectors

$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$$

Trigonometric Form and Roots of Complex Numbers

$$r_1(\cos\theta_1 + i\sin\theta_1) \bullet r_2(\cos\theta_2 + i\sin\theta_2) = r_1 r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$$

$$\frac{r_1\left(\cos\theta_1 + i\sin\theta_1\right)}{r_2\left(\cos\theta_2 + i\sin\theta_2\right)} = \frac{r_1}{r_2}\left[\cos\left(\theta_1-\theta_2\right) + i\sin\left(\theta_1-\theta_2\right)\right]$$

$$\left[r\left(\cos\theta + i\sin\theta\right)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

$$r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n} + k\bullet\frac{360^\circ}{n}\right) + i\sin\left(\frac{\theta}{n} + k\bullet\frac{360^\circ}{n}\right)\right]$$

Parabola

$$(x-h)^2 = 4p(y-k), \quad (h,k), \quad (h,k+p), \quad y = k-p$$

$$(y-k)^2 = 4p(x-h), \quad (h,k), \quad (h+p,k), \quad x = h-p$$

Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Ellipse

$$\frac{\left(x-h\right)^2}{a^2} + \frac{\left(y-k\right)^2}{b^2} = 1, \quad (h,k), \quad (h\pm a,k),$$

$(h\pm c,k)$ and $c^2 = a^2 - b^2$

$$\frac{\left(x-h\right)^2}{b^2} + \frac{\left(y-k\right)^2}{a^2} = 1, \quad (h,k), \quad (h,k\pm a),$$

$(h,k\pm c)$ and $c^2 = a^2 - b^2$

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1, \quad (h,k), \quad (h\pm a,k),$$

$(h\pm c,k)$, $\quad y = \pm\frac{b}{a}(x-h)+k$ and $c^2 = a^2 + b^2$

$$\frac{\left(y-k\right)^2}{a^2} - \frac{\left(x-h\right)^2}{b^2} = 1, \quad (h,k), \quad (h,k\pm a),$$

$(h,k\pm c)$, $\quad y = \pm\frac{a}{b}(x-h)+k$ and $c^2 = a^2 + b^2$

Arithmetic Sequences and Series

$$a_n = a_1 + (n-1)d \qquad d = a_{n+1} - a_n$$

$$S_n = \frac{n}{2}\left(a_1 + a_n\right)$$

Geometric Sequences and Series

$$a_n = a_1 r^{n-1} \qquad r = \frac{a_{n+1}}{a_n}$$

$$S_n = a_1\frac{\left(1-r^n\right)}{1-r}$$

$$S_\infty = \frac{a_1}{1-r} \quad \text{when } |r| < 1$$

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Formulas from MATH 1710:

Quadratic Equations

1. Vertex: $(h,k) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
2. Vertex form of a Parabola: $y = a(x-h)^2 + k$

Properties of Logarithms

1. $\log_a x = k$ is equivalent to $x = a^k$
2. $\log_a M + \log_a N = \log_a(M \cdot N)$
3. $\log_a M - \log_a N = \log_a\left(\frac{M}{N}\right)$
4. $\log_a(M^k) = k \log_a M$
5. Change of base: $\log_a x = \frac{\log_b x}{\log_b a}$

Formulas from Geometry

1. Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
3. Equation of a Circle: $(x-h)^2 + (y-k)^2 = r^2$

Interest Formulas

1. Simple Interest: $A = P(1 + rt)$
2. Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
3. Continuous Interest: $A = Pe^{rt}$